Laboratory calibration of a MEMS accelerometer sensor

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General Idea

- We express any acceleration that acts on the device as a function
$$|a| = a(s_1, s_2, s_3, b_1, b_2, b_3, \phi, \psi, \theta)$$
- s_1, s_2, s_3 – shifts
- b_1, b_2, b_3 – scaling coefficients
- ϕ, ψ, θ – angles between the axis of the accelerometers
- When the device stands still, then the only acceleration it measures should be due to gravitational force.
- We make a least squares fit to determine the coefficients, using the fact, that
$$|a| = g$$

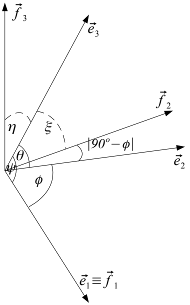
Calibration

- $\hat{a}_x$, $\hat{a}_y$ and $\hat{a}_z$ – measured values
- a_x, a_y and a_z – calibrated values
- In many papers it is suggested that a linear connection between the measured and the true acceleration is valid:

$$a_x = (\hat{a}_x - s_1)/b_1$$
$$a_y = (\hat{a}_y - s_2)/b_2$$
$$a_z = (\hat{a}_z - s_3)/b_3$$

- Let the coordinates of the acceleration vector in an orthonormal coordinate system be:

$$\overline{\boldsymbol{a}}^{orth} := \begin{bmatrix} \overline{a}_x^{orth} \\ \overline{a}_y^{orth} \\ \overline{a}_z^{orth} \end{bmatrix}$$



- We obtain the expression

$$\overline{\boldsymbol{a}}^{orth} = \overline{T}^{orth}\overline{T}\,T\widehat{\boldsymbol{a}} - \overline{T}^{orth}\overline{T}\boldsymbol{s}$$

$$\overline{T}^{orth} = \begin{bmatrix} 1 & \cos\phi & \cos\psi \\ 0 & \sin\phi & \dfrac{\cos\theta - \cos\phi\cos\psi}{\sin\phi} \\ 0 & 0 & \dfrac{\sqrt{1-\cos^2\phi - \cos^2\psi - \cos^2\theta + 2\cos\phi\cos\psi\cos\theta}}{\sin\phi} \end{bmatrix}$$

$$\overline{T} = \frac{1}{den} \begin{bmatrix} -\sin^2\theta & \cos\phi - \cos\theta\cos\psi & \cos\psi - \cos\phi\cos\theta \\ \cos\phi - \cos\theta\cos\psi & -\sin^2\psi & \cos\theta - \cos\phi\cos\psi \\ \cos\psi - \cos\phi\cos\theta & \cos\theta - \cos\phi\cos\psi & -\sin^2\phi \end{bmatrix}$$

$$den = -1 + \cos^2\phi + \cos^2\psi + \cos^2\theta - 2\cos\phi\cos\psi\cos\theta$$

$$\boldsymbol{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \quad T = \begin{bmatrix} 1/b_1 & 0 & 0 \\ 0 & 1/b_2 & 0 \\ 0 & 0 & 1/b_3 \end{bmatrix}$$

Estimation of the error due to non-orthogonality

- We want to evaluate what is the error that can be caused by not considering the non-orthogonality of the axis.
- It holds that

$$max|a(a_x, a_y, a_z, \phi, \psi, \theta) - a(a_x, a_y, a_z, \pi/2, \pi/2, \pi/2)| \approx 0.312817,$$

where the maximum is taken over all $0.98\pi \leq \phi, \psi, \theta \leq 1.02\pi$ and a_x, a_y and a_z satisfy the condition

$$a(a_x, a_y, a_z, \phi, \psi, \theta) = g$$

- This error leads to an error when estimating the position after 10s of integrating of more than 300m!

Tests

- We are given 3 sets with data (from 3 different phones), measured from 27 different positions of the phone
- We have made tests, calibrating the accelerometers using data from 20 different positions of the phone.
- We then check the results by comparing the calibrated acceleration (with the evaluated parameters) with 7 more positions of the phone.

Dataset 1

- Using positions 1–20, we obtain

$$s_1 \to 0.318321, s_2 \to 0.322794, s_3 \to -1.09059,$$
$$b_1 \to -1.00472, b_2 \to -1.00124, b_3 \to -0.989255,$$
$$\phi \to 1.56932, \psi \to 1.5709, \theta \to 1.57296$$

- Using positions 5–25:

$$s_1 \to 0.317766, s_2 \to 0.320884, s_3 \to -1.09105,$$
$$b_1 \to -1.00465, b_2 \to -1.00117, b_3 \to -0.989277,$$
$$\phi \to 1.57035, \psi \to 1.57078, \theta \to 1.57296$$

- The relative error $\left|\frac{a-g}{g}\right|$ is less than 0.21%.

Dataset 2

- Using positions 1–20, we obtain

$$s_1 \to -0.476512, s_2 \to 0.615683, s_3 \to 0.206696,$$
$$b_1 \to -0.998876, b_2 \to -0.989542, b_3 \to -1.01193,$$
$$\phi \to 1.56193, \psi \to 1.56799, \theta \to 1.57221$$

- Using positions 5–25:

$$s_1 \to -0.469784, s_2 \to 0.608425, s_3 \to 0.208206,$$
$$b_1 \to -0.999573, b_2 \to -0.989443, b_3 \to -1.01176,$$
$$\phi \to 1.56219, \psi \to 1.56969, \theta \to 1.57057$$

- Relative error: less than 0.45%.

Dataset 3

- The measured data in positions 3,21 is respectively:

$$10.20588166, 0.146627372, 0.293913142$$
$$10.18267071, 0.144573621, 0.312617999$$

- Using positions 1–20, we obtain a relative error of 0.26%
- Using positions 3–23, we obtain a relative error of 0.98%
- Therefore, positions should be chosen as independent as possible or the data should be processed before calibration

THANK YOU FOR YOUR ATTENTION!