Laboratory calibration of a MEMS accelerometer sensor

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Problem

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We consider only the stationary case: a constant acceleration a applies to the phone; its sensor outputs the measurements m.

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We consider only the stationary case: a constant acceleration a applies to the phone; its sensor outputs the measurements m.

In practice, $m \neq a$ due to

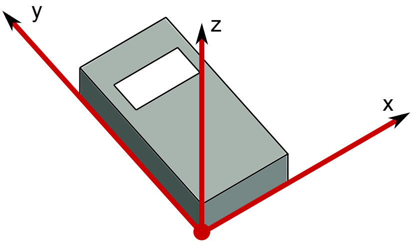
- scaling errors,
- crosstalk,
- offset errors,
- non-orthogonality of the accelerometer axes,
- ...

We want to find and calibrate a model which allows us to reconstruct

$$m \mapsto a.$$

Problem

We fix a coordinate system relative to the phone:



All accelerations acting on the phone

$$a = (a_x, a_y, a_z)$$

are considered in this coordinate system.

It may not be aligned with the axes of the accelerometer!

Mathematical model

For an acceleration $a \in \mathbb{R}^3$ acting on the phone, we obtain a measurement $m \in \mathbb{R}^3$ from the accelerometer.

We assume a simple linear model:

$$m = F(a) = Xa + y, \qquad X \in \mathbb{R}^{3\times 3}, y \in \mathbb{R}^3.$$

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In this model, we can treat

- scaling errors (diag(X))
- crosstalk (off-diagonal entries of X)
- offset errors (y)
- a sensor which is installed in a rotated way with respect to the phone (X a rotation matrix)
- combinations of all these effects

Mathematical model

Once we have determined the parameters X and y, we can easily invert the model to obtain

$$a = F^{-1}(m) = X^{-1}(m - y).$$

Thus we can get the (approximate) true accelerations a for any measurement m.

Determining the parameters

Assume we have put the phone in a reference position where we know the acceleration *a* which acts on it. We take the measurement *m* of the accelerometer. Due to the model

$$Xa + y = m,$$

we have

$$\sum_{j=1}^{3} x_{ij} a_j + y_i = m_i \qquad i = 1, 2, 3.$$

System of 3 linear equations – but the matrix is unknown!

Determining the parameters

Collecting all unknowns X and y into a vector

$$z = (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, y_1, y_2, y_3)^\top,$$

we can write the three equations

$$\sum_{j=1}^{3} x_{ij} a_j + y_i = m_i \qquad i = 1, 2, 3.$$

as

$$\begin{pmatrix} a_1 & a_2 & a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & a_1 & a_2 & a_3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_1 & a_2 & a_3 & 0 & 0 & 1 \end{pmatrix} z = m.$$

Denote this matrix by $K(a) \in \mathbb{R}^{3\times 3}$ so that $K(a)z = m$.

Determining the parameters

The previous system had 12 unknowns, 3 equations.

We take multiple measurements $m^{(i)}$, $i = 1, \ldots, N$ in reference positions with known accelerations $a^{(i)}$. Then we get the $3N \times 12$ block system

$$Kz := \begin{bmatrix} K(a^{(1)}) \\ K(a^{(2)}) \\ \vdots \\ K(a^{(N)}) \end{bmatrix} z = \begin{bmatrix} m^{(1)} \\ m^{(2)} \\ \vdots \\ m^{(N)} \end{bmatrix} =: \overline{m}.$$

Determining the parameters

We have $K \in \mathbb{R}^{3N\times 12}$. If $N = 4$ and the 4 vectors

$$\left(a_1^{(i)}, a_2^{(i)}, a_3^{(i)}, 1\right), \qquad i = 1, 2, 3, 4$$

are linearly independent, then there is a unique solution.

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In practice, one can use more measurements, say $N = 6$. If the above condition is fulfilled, the linear least-squares problem

$$K^\top K z = K^\top \overline{m}$$

gives us the unique minimizer of

$$|\overline{m} - Kz|^2 .$$

Determining the parameters

After solving the least-squares problem for z, we obtain the coefficients of X and y from

$$z = (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, y_1, y_2, y_3)^\top.$$

The device is calibrated and for every measurement m we obtain the acceleration a from

$$a = X^{-1}(m - y).$$

Calibration procedure

For the calibration procedure, the user needs:

- a table (optional)
- a window

We take measurements in 6 different positions. In each position, we measure for 5 seconds and take the average of the sensor data over this time to reduce the influence of time-dependent noise.

Calibration procedure: step 1

$$a^{(1)} = (0, 0, -g)$$

measure for 5 seconds, average the measurements $\rightarrow m^{(1)}$

Calibration procedure: step 2

$$a^{(2)} = (0, 0, \mathrm{g})$$

measure for 5 seconds, average the measurements $\rightarrow m^{(2)}$

Calibration procedure: step 3

$$a^{(3)} = (0, -g, 0)$$

measure for 5 seconds, average the measurements $\rightarrow m^{(3)}$

Calibration procedure: step 4

$$a^{(4)} = (0, \mathrm{g}, 0)$$

measure for 5 seconds, average the measurements $\rightarrow m^{(4)}$

Calibration procedure: step 5

$$a^{(5)} = (-g, 0, 0)$$

measure for 5 seconds, average the measurements $\rightarrow m^{(5)}$

Calibration procedure: step 6

$$a^{(6)} = (g, 0, 0)$$

measure for 5 seconds, average the measurements $\rightarrow m^{(6)}$

Simplification of the least-squares problem

Recall that

$$K = \begin{bmatrix} K(a^{(1)}) \\ \vdots \\ K(a^{(N)}) \end{bmatrix},$$

$$K(a) = \begin{pmatrix} a_1 & a_2 & a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & a_1 & a_2 & a_3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_1 & a_2 & a_3 & 0 & 0 & 1 \end{pmatrix}$$

and we need to solve the linear least-squares problem

$$\underbrace{K^\top K}_{\in \mathbb{R}^{12 \times 12}} z = K^\top \overline{m}.$$

Simplification of the least-squares problem

For the special choice in our calibration procedure

$$\begin{aligned} a^{(1)} &= (-g, 0, 0), \quad & a^{(3)} &= (0, -g, 0), \quad & a^{(5)} &= (0, 0, -g), \\ a^{(2)} &= (g, 0, 0), & a^{(4)} &= (0, g, 0), & a^{(6)} &= (0, 0, g), \end{aligned}$$

it is easy to show that

$$K^\top K = \mathrm{diag}(2g^2, 2g^2, 2g^2, 2g^2, 2g^2, 2g^2, 2g^2, 2g^2, 2g^2, 6, 6, 6).$$

Simplification of the least-squares problem

Therefore, in this case the least-squares problem has the closed-form solution

$$X = \frac{1}{2g} \begin{pmatrix} -m_1^{(1)} + m_1^{(2)} & -m_1^{(3)} + m_1^{(4)} & -m_1^{(5)} + m_1^{(6)} \\ -m_2^{(1)} + m_2^{(2)} & -m_2^{(3)} + m_2^{(4)} & -m_2^{(5)} + m_2^{(6)} \\ -m_3^{(1)} + m_3^{(2)} & -m_3^{(3)} + m_3^{(4)} & -m_3^{(5)} + m_3^{(6)} \end{pmatrix},$$

$$y = \frac{1}{6} \sum_{i=1}^{6} m^{(i)}.$$

No matrix inversion needed anymore for calibration!
(Can be done very quickly on the phone itself.)

Example: Huawei

We have 29 accelerations $m^{(i)}$ measured in different positions.

$$\begin{matrix}
-9.54983 & 0.37829 & -0.999283 \\
10.145 & 0.314528 & -1.37789 \\
0.413309 & -9.47641 & -1.62076 \\
0.368766 & 10.1442 & -0.910232 \\
0.510292 & 0.338026 & -10.7851 \\
0.231834 & 0.482186 & 8.60553 \\
0.230219 & 7.85729 & 5.09836 \\
0.304598 & 8.0554 & -7.09352 \\
-0.612011 & -7.07391 & -7.39908 \\
-0.741095 & -7.0657 & 5.22667 \\
8.34517 & -1.00171 & 4.35023 \\
7.90356 & -1.01121 & -7.15935 \\
-7.21644 & -0.678396 & 5.06995 \\
\vdots & \vdots & \vdots
\end{matrix}$$

Example: Huawei

The first $N = 6$ measurements were taken in the 6 reference positions, and therefore we know the exact accelerations $a^{(i)}$, $i = 1, \ldots, 6$:

$$
\begin{array}{l}
(-g \quad 0 \quad 0) \\
(\ \ g \quad 0 \quad 0) \\
(\ \ 0 \quad -g \quad 0) \\
(\ \ 0 \quad g \quad 0) \\
(\ \ 0 \quad 0 \quad -g) \\
(\ \ 0 \quad 0 \quad g)
\end{array}
$$

Example: Huawei

After running our algorithm, we obtain the model parameters

$$X = \begin{pmatrix} 1.00381 & -0.00227028 & -0.0141925 \\ -0.00324982 & 1.00003 & 0.00734762 \\ -0.019297 & 0.0362144 & 0.988311 \end{pmatrix},$$

$$y = \begin{pmatrix} 0.353222 \\ 0.363473 \\ -1.18129 \end{pmatrix}.$$

Offsets are within the tolerances from the sensor data sheet:
± 0.49 m/s^2 for x and y axis
± 1.96 m/s^2 for z axis

Example: Huawei

We check how well the linear model fits the calibration dataset.

$$\max_{i=1,\dots,6} \frac{\left\| F^{-1}(m^{(i)}) - a^{(i)} \right\|}{\left\| a^{(i)} \right\|} = 0.010601$$

Errors in original sensor data:

$$\max_{i=1,\dots,6} \frac{\left\| m^{(i)} - a^{(i)} \right\|}{\left\| a^{(i)} \right\|} = 0.17386$$

Improvement: 16.4x

Example: Huawei

After calibrating using the first 6 datasets, we check the norms of the reconstructed accelerations to see if they match the expected length g.

$$\max_{i=1,\dots,29} \frac{\big|\|F^{-1}(m^{(i)})\| - \mathrm{g}\big|}{\mathrm{g}} = 0.0315425$$

The original sensor data was much worse:

$$\max_{i=1,\dots,29} \frac{\big|\|m^{(i)}\| - \mathrm{g}\big|}{\mathrm{g}} = 0.122075$$

Improvement: 3.87x

Example: GT

For a second dataset ("GT"), we have 27 measurements and use the first 6 to calibrate the device.

$$X = \begin{pmatrix} 0.990908 & 0.0193084 & -0.0191228 \\ -0.000163521 & 0.981047 & -0.00689508 \\ 0.0228047 & 0.00124993 & 0.998542 \end{pmatrix},$$

$$y = \begin{pmatrix} 0.482181 \\ 0.0587712 \\ 0.0440956 \end{pmatrix}.$$

Example: GT

Calibration error:

$$\max_{i=1,\ldots,6} \frac{\left\| F^{-1}(m^{(i)}) - a^{(i)} \right\|}{\left\| a^{(i)} \right\|} = 0.0158458$$

Errors in original sensor data:

$$\max_{i=1,\ldots,6} \frac{\left\| m^{(i)} - a^{(i)} \right\|}{\left\| a^{(i)} \right\|} = 0.0725016$$

Improvement: 4.58x

Example: GT

Again we check the norms:

$$\max_{i=1,\dots,27} \frac{\left|\|F^{-1}(m^{(i)})\| - \mathrm{g}\right|}{\mathrm{g}} = 0.0138985$$

Original sensor data:

$$\max_{i=1,\dots,27} \frac{\left|\|m^{(i)}\| - \mathrm{g}\right|}{\mathrm{g}} = 0.0582853$$

Improvement: 4.19x

Test of robustness of calibration procedure

Suggested by industry partner: compare results of several calibrations.

We made 20 measurements of 5 seconds each in each of the 6 positions with another Huawei phone. Mean over the 20 trials:

$$\begin{pmatrix} 1.00367 & -0.00200432 & 0.0211193 \\ 0.00396457 & 1.00055 & -0.00489847 \\ 0.00346578 & 0.00534244 & 0.989011 \end{pmatrix}, \quad \begin{pmatrix} 0.264937 \\ 0.418299 \\ -1.1462 \end{pmatrix}$$

Standard deviations:

$$\begin{pmatrix} 0.00021274 & 0.0016114 & 0.000203203 \\ 0.00109465 & 0.000171741 & 0.000251313 \\ 0.000307081 & 0.000715155 & 0.000315258 \end{pmatrix}, \quad \begin{pmatrix} 0.00439571 \\ 0.00300421 \\ 0.00463018 \end{pmatrix}$$

Conclusions

We have presented a calibration method which

- is easy to implement
- requires only simple measurements which the end user can perform in a few minutes using no extra tools
- is computationally very cheap
- significantly reduces the error on the tested data sets
- can be applied in the non-stationary case

Thank you for your attention!